How to Find Remaining Zeroes

Q. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find all zeroes.

Solution: Let $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Since two zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$ of p(x)

$\therefore$ $[x-(2 + \sqrt{3})]$ and $[x- (2 - \sqrt{3})]$ are the factors of p(x)

$= [x-(2 + \sqrt{3})]\ [x- (2 - \sqrt{3})]$

$= (x-2- \sqrt{3})(x-2 + \sqrt{3})$

$= (x-2)^2 - (\sqrt{3})^2$

$= x^2-4x+4-3 = x^2 - 4x + 1$

$x^2 - 4x + 1$ is a factor of the given polynomial p(x)

Now, we divide the given polynomial by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

By Euclid's Division algorithm

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)\ (x^2 - 2x - 35) + 0$

$p(x) = (x^2 - 4x + 1)\ (x^2 - 2x - 35)$

$= (x^2 - 4x + 1)\ (x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)\ [x(x- 7) + 5\ (x-7)]$

$= (x^2 - 4x + 1)\ (x - 7)\ (x + 5)$

Hence, all zeroes of the given polynomial p(x) are $2 - \sqrt{3}$, $2 - \sqrt{3}$, 7 and -5. *Answer*

Do Your Self

1. Obtain all the zeroes of $3x^4+6x^3-2x^2-10x-5$ if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.
2. Find all the zeros of $(x^4 + x^3 - 23x^2 - 3x + 60)$, if it is given that two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$.
3. Find all the zeros of $(x^4 + 2x^3 - 13x^2 - 12x + 21)$, if it is given that two of its zeros are $2+\sqrt{3}$ and $2 - \sqrt{3}$.

Q. If the zeroes of polynomial $p(x) = x^3 - 3x^2 + x + 1$ zeroes are $a - b, a, a + b$, find a and b.

Given $p(x) = x^3 - 3x^2 + x + 1$ & zeroes are $a - b, a, a + b$

Then $\alpha = a - b$, $\beta = a$ and $\gamma = a + b$.

$\therefore$ **Sum of zeroes** $= \alpha + \beta + \gamma = \frac{-B}{A}$

$\Rightarrow (a - b) + a + (a + b) = \frac{-(-3)}{1}$

$\Rightarrow (a - b) + a + (a + b) = 3$

$\Rightarrow a - b + a + a + b = 3$

$\Rightarrow 3a = 3$

$\therefore a = \frac{3}{3} = 1$ ...(i)

Product of zeroes $= \alpha\beta\gamma = \frac{-D}{A}$

$\Rightarrow (a - b)\, a\, (a + b) = \frac{-1}{1}$

$\Rightarrow (a - b)\, a\, (a + b) = -1$

$\Rightarrow (a^2 - b^2)a = -1$

$\Rightarrow a^3 - ab^2 = -1$ ... (ii)

Putting the value of a from equation (i)

$\therefore (1)^3 - (1)b^2 = -1$

$\Rightarrow 1 - b^2 = -1$

$\Rightarrow -b^2 = -1 - 1$

$\Rightarrow b^2 = 2$

$\therefore b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \pm\sqrt{2}$. *Answer*

Do Your Self

1) **If the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a + b$ and $a + 2b$ for some real numbers a and b,**
2) **If $(a - b)$, a and $(a + b)$ are zeros of the polynomial $2x^3 - 6x^2 + 5x - 7$, write the value of a and b.**